A New Strategyproof Greedy-Allocation Combinatorial Auction Protocol and its Extension to Open Ascending Auction Protocol

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Abstract

This paper proposes a new combinatorial auction protocol called Average-Max-Minimal-Bundle (AM-MB) protocol. The characteristics of the AM-MB protocol are as follows: (i) it is strategyproof, i.e., truth-telling is a dominant strategy, (ii) the computational overhead is very low, since it allocates bundles greedily thereby avoiding an explicit combinatorial optimization problem, and (iii) it can obtain higher social surplus and revenue than can the Max-Minimal-Bundle (M-MB) protocol, which also satisfies (i) and (ii). Furthermore, this paper extends the AM-MB protocol to an open ascending-price protocol in which straightforward bidding is an ex-post Nash equilibrium.

Introduction

Computational mechanism design (Dash, Jennings, & Parkes 2003) has recently attracted much attention. In particular, auction mechanisms provide a convenient way to realize an efficient allocation. This paper proposes a new combinatorial auction protocol called Average-Max-Minimal-Bundle (AM-MB). The paper then extends the AM-MB protocol to an open-ascending price combinatorial auction. The AM-MB protocol is a combinatorial auction that satisfies the condition of strategyproofness and requires very little computational overhead since it does not solve a combinatorial optimization problem. In the AM-MB based open-ascending price combinatorial auction, sincere bidding is ex-post a Nash equilibrium, and communication and computational costs are low.

The AM-MB protocol is strategyproof. Namely, truth-telling is a dominant strategy since this protocol is an instance of a PORF protocol. The PORF (Price-Oriented Rationing Free) protocol is a generic combinatorial auction protocol that satisfies the conditions for strategyproofness (Yokoo 2003). PORF is one of frameworks for designing auction protocols. There have been several frameworks, e.g., (Myerson 1981)(Gonen, Bartal, & Nisan 2003)(Lavi, Mu'alem, & Nisan 2003), that account for the strategyproof. An important feature of PORF is that allocations are computed greedily, which generally leads to very low computational costs[1].

The AM-MB protocol benefits from the same computational expediency. In contrast, consider the well-known Vickrey-Clarke-Groves (VCG) mechanism, applied to a combinatorial auction setting. The mechanism requires the solution to an explicit combinatorial optimization problem (as part of the winner determination problem). Indeed, VCG actually requires that we solve this difficult problem several times for a single pricing calculation. For one pricing calculation for n winners, we need $n+1$ winner determination calculations in VCG[2].

The AM-MB protocol can obtain better social surplus and revenue compared to an existing combinatorial auction protocol called Max-Minimal-Bundle (M-MB) (Yokoo 2003) protocol. The M-MB protocol satisfies the very desirable false-name-proofness property. On the other hand, it suffers from the possibility that the generated social surplus will be very low. This is because the M-MB protocol uses the minimal bundles' maximum valuations to calculate payments. Therefore, the payments tend to set at an unnecessarily high-level. As a result, there may be bidders who may value an item most and yet not be able to afford it. In the AM-MB protocol, we propose a new pricing scheme that focuses on the price of each item, not that of a bundle.

We extend the AM-MB protocol to an open-format protocol; in the resulting open ascending-price protocol, straightforward bidding is an ex-post Nash equilibrium. Open format auctions such as English, Dutch, and Ausubel types are said to outperform sealed-bid format auctions (Ausubel 2002) in practice. For example, the VCG auction is clearly not as popular as the ubiquitous English auction, though the outcomes of these two auction formats are identical. Ausubel argued that simplicity and privacy-preservation seem to encourage more bidders to bid honestly and to provide the seller with a higher revenue (Ausubel 2002). This simplicity allows bidders to understand the Ausubel protocol more easily than the VCG auction. The privacy preservation of the bidders' values encourages bidders to join auctions, since they only need to reveal only parts of their demand

[1] Another important feature of PORF is false-name-proofness(Yokoo 2003).

[2] In the single-good multiple unit auction, the paper(Kothari, Parkes, & Suri 2003) shows a polynomial-time approximation scheme. In this paper, we handle multiple-goods auctions.

curve. On the other hand, in sealed-bid formats, participants may not be very comfortable about truthfully revealing their entire set of private values, even though doing so is a dominant strategy.

The above studies handle open ascending-price "multi-unit" auctions. On the other hand, we proposed an open ascending-price combinatorial auction based on AM-MB. The proposed open ascending-price combinatorial auction protocol is designed by applying the concept of options. An option is the right to buy an item at the price announced at the time the option is offered. This paper proves that the prices obtained by our new open ascending-price combinatorial protocol converge to the price obtained by the AM-MB protocol. This means that the proposed open ascending auction offers the same features in terms of social surplus and seller's revenue. Furthermore, this paper proves that straightforward bidding is an ex-post Nash equilibrium in the proposed open ascending auction.

Another advantage of the proposed open ascending-price combinatorial auction is its low communication cost. In the protocol, only a single price is announced to bidders in each round, and each bidder declares only a set of items that he/she might be interested in buying. In the protocol, bidders do not need to declare 2^m evaluation values as in many other combinatorial auction schemes, e.g., VCG. In the proposed open ascending auction, bidders need to submit their demands on combinations of goods only once.

The rest of the paper is organized as follows. First, the next section provides definitions of some basic terms throughout the paper as well as an explanation of the the Price-Oriented, Rationing-Free (PORF) protocol. In the following section, this paper introduces the AM-MB protocol, a one-shot sealed bid auction based on the PORF protocol. Third, the AM-MB protocol is extended to an open ascending protocol that employs the AM-MB payment scheme. The important features of our protocol are then proved.

Preliminaries

Problem Settings

This section gives the definition of the domain model. A set of bidders is $N = \{1, 2, \ldots, n\}$. A set of items is $M = \{1, 2, \ldots, m\}$. Each bidder i has his/her preferences for each bundle $B \subseteq M$. $v(B, \theta_i)$ is bidder i's evaluation value for bundle B. Here, θ_i is i's true type. w_B represents the relative weight of bundle B. In this paper, for simplicity, we assume $w_B = |B|$. However, we can generalize the AM-MB protocol in the case where the relative weights of items/bundles can be different. We assume a quasi-linear, private value model with no allocative externality. Further, we assume free disposal.

Next, the concept of the minimal bundle is defined.

Definition 1 (Minimal bundle) *Bundle B is called minimal for bidder i if for all $B' \subset B$ and $v(B', \theta_i) < v(B, \theta_i)$ holds.*

For example, suppose there are 3 items: 1, 2, and 3. If bidder 1 wants to obtain items in two distinct bundles, he tries to get the two bundles {1,2} and {3}. If bidder 2 wants to obtain items in a single bundle, she tries to get just one bundle, e.g., {1,2}. Here, bidder 1's minimal bundles are {1,2} and {3}, while bidder 2's minimal bundle is {1,2}.

PORF

The PORF (Price-Oriented Rationing Free) protocol is a generic combinatorial auction protocol that satisfies the conditions for strategyproofness and false-name proofness (Yokoo 2003). An outline of the PORF protocol can be described as follows: (1) for each bidder, the payment of each bundle of items is determined independently of his/her own declaration, (2) the protocol allocates each bidder a bundle that maximizes his/her utility independently of the allocations of other bidders, i.e., it's rationing free. The PORF protocol is defined as follows.

Definition 2 (PORF protocol) *Each bidder i declares his/her type $\tilde{\theta}_i$, which is not necessarily the true type θ_i. For each bidder i and for each bundle $B \subseteq M$, the payment $p_{po}(B, i)$ is determined. This payment must be determined independently of i's declared type $\tilde{\theta}_i$, but it might be dependent on the declared types of the other bidders. We assume $p_{po}(\emptyset, i) = 0$ holds. Also, if $B \subseteq B'$, then $p_{po}(B, i) \leq p_{po}(B', i)$ holds. For bidder i, a bundle B^* is allocated where $B^* = \arg\max_{B \subseteq M} v(B, \tilde{\theta}_i) - p_{po}(B, i)$. Bidder i pays $p_{po}(B^*, i)$. If multiple bundles exist that maximize i's utility, one of these bundles is allocated.*

Definition 3 (Allocation feasibility) *The allocation must be feasible, i.e., for bundles B_i and B_j allocated to bidders i and j respectively, $B_i \cap B_j \neq \emptyset$ holds. Thus, we show this for the AM-MB protocol in Theorem1.*

A PORF protocol is strategyproof since bidder i's payment is determined independently of i's declared type, and he/she can obtain the bundle that maximizes his/her utility independently of the allocations of other bidders. As a result, allocation feasibility is satisfied. In this way, the PORF protocol is rationing-free(Yokoo 2003).

Max-Minimal-Bundle (M-MB) Protocol

The M-MB protocol is an instance of the PORF protocol, proposed by (Yokoo 2003). In the M-MB, the bidder i' s payment for bundle B is defined as follows:

- $p_{po}(B, i) = \max_{B_j \subseteq M, j \neq i} v(B_j, \theta_j)$, where $B \cap B_j \neq \emptyset$ and B_j is minimal for bidder j.

Namely, the payment for bundle B is equal to the highest evaluation value of a bundle that is minimal and intersects with the items of bundle B. This pricing scheme satisfy allocation feasibility (Yokoo 2003).

Average-Max-Minimal-Bundle (AM-MB) Protocol

In this section we discuss the design of the Average-Max-Minimal-Bundle (AM-MB) Protocol. This is also an instance of the PORF protocol, so it satisfies strategyproof property. Consequently, here we define the protocol by using true type θ_j. In the AM-MB Protocol, the bidder i's payment for bundle B is defined as follows:

Definition 4 (AM-MB price) $p_{po}(B, i) = w_B \max_{B_j \subseteq M, j \neq i} v(B_j, \theta_j)/w_{B_j}$, *where* $B \cap B_j \neq \emptyset$ *and* B_j *is minimal for bidder* j. w_B *is the size of bundle* B.

Namely, the payment for bundle B is equal to the size times the highest evaluation value of an item, which intersects with the items of bundle B.

Definition 5 (AM-MB protocol) *The AM-MB protocol is a PORF protocol (in the definition 2) that instantiates the payment* $p_{po}(B, I)$ *as the AM-MB payment in the definition 4.*

Theorem 1 *AM-MB protocol satisfies allocation feasibility.*

We omit the proof due to space limitation.

We can consider the AM-MB protocol as a kind of greedy allocation protocol, i.e., for each item l, only the bidder i who has the highest average value $v(i, B_i)/w_{B_i}$, where $l \in B_i$, has the right to obtain l.

The AM-MB protocol is a greedy allocation protocol. As a result the computation required to execute this protocol is very low compared to VCG. In VCG, we need to solve a combinatorial optimization problem for winner determination. In fact, we need to solve the winner determination problem n+1 times: n times with a single player (who is a winner) missing from the problem, and once with all players (who are winners) considered. Even for the worst case in the AM-MB protocol, where every bundle is minimal for every bidder, the AM-MB protocol does not compute a combinatorial optimization problem.

The AM-MB protocol can obtain better social surplus and revenue compared to M-MB. We show this in Section 5. The AM-MB protocol cannot satisfy the condition of being false-name-proof, although the M-MB protocol is false-name-proof.

Open Ascending-price Combinatorial Auction based on AM-MB Protocol

Baseline Price and Bundle Price

In this section, we design the open ascending-price combinatorial auction based on the AM-MB protocol. First, we introduce the baseline price and the bundle price. Second, we present an example. Finally, we show the proposed protocol. The baseline price is defined for each item. The baseline price is employed for the ascending auction to periodically increase prices for each item. The bundle price is defined for each bundle. At most one bidder can receive a positive benefit when purchasing the bundle at the bundle price.

Definition 6 (Baseline price) *For item* l, $(j^*, B^*) = \arg\max_{j^* \in N, B^* \ni l} v(B^*, \theta_j)/w_{B^*}$, *where* B^* *is a minimal bundle. The baseline price for item* l *is* $p_{base}(l) = \max_{j \neq j^*, B \ni l} v(B, \theta_j)/w_B$, *where B is a minimal bundle.*

Essentially, the baseline price is the second-highest average value.

By using the baseline price, the bundle price for bundle B is defined as follows:

Definition 7 (Bundle price) $p(B) = w_B \max_{l \in B} p_{base}(l)$

Prices are non-linear (being defined for each bundle) and anonymous (meaning the price is the same for all bidders).

Theorem 2 *When applying the bundle price, for each bundle there is at most one bidder who can receive a positive utility.*

We omit the proof due to space limitation.

Theorem 3 *The allocation result of the AM-MB Protocol is equivalent to the allocation result based on the bundle price.*

Proof: In this proof, we show the followings:

(1)In the AM-MB protocol, if bidder i's utility in obtaining bundle B is non-negative, i.e., $v(B, \theta_i) \geq p(B, i)$, then, $p(B, i)$ is equal to the bundle price $p(B)$. Thus, in either scheme, bidder i will obtain B if it maximizes bidder i's utility.

(2) In the AM-MB protocol, if bidder i's utility in obtaining bundle B is negative, i.e., $v(B, \theta_i) < p(B, i)$, then, the bundle price $p(B)$ is more than $v(B, \theta_i)$. Thus, in either scheme, bidder i is not willing to obtain B.

In terms of (1), from the precondition, $v(B, \theta_i) \geq p(B, i) = w_B \max_{j \neq i, B'} v(B', \theta_j)/w_{B'}$, where B' conflicts with B and is minimal for bidder j. Therefore, for each $j \neq i$, $v(B, \theta_i)/w_B \geq v(B', \theta_j)/w_{B'}$, where B' conflicts with B and is minimal for bidder j. This equation means that for each item $l \in B$, i has the highest average evaluation value, i.e., $j^* = i$. Thus, $p_{base}(l) = \max_{j \neq i, B' \ni l} v(B', \theta_j)/w_{B'}$, where B' is a minimal bundle. The definition of $p(B)$ is $w_B \max_{l \in B} p_{base}(l)$. Thus, $p(B) = w_B \max_{l \in B} \max_{j \neq i, B' \ni l} v(B', \theta_j)/w_{B'}$, where B' is a minimal bundle.

We can omit l form the right side of this equation since this equation holds for all $l \in B$. Thus we obtain $w_B \max_{j \neq i, B'} v(B', \theta_j)/w_{B'}$, where B' is a minimal bundle and conflicts with B. This is identical to the definition of $p(B, i)$. Thus, $p(B, i) = p(B)$ holds.

In terms of (2), from the precondition, we obtain $v(B, \theta_i) < p(B, i) = w_B \max_{j \neq i, B'} v(B', \theta_j)/w_{B'}$, where B' conflicts with B and is minimal. Therefore, for some j, B', where $j \neq i$ and B' conflicts with B, $v(B, \theta_i)/w_B < v(B', \theta_j)/w_{B'}$ holds. If B conflicts with B' over the item l, $p_{base}(l) \geq v(B', \theta_j)/w_{B'} > v(B, \theta_i)/w_B$holds. Therefore, $p(B) \geq w_B p_{base}(l) > v(B, \theta_i)$ holds. □

Example

Let us provide an example. First, we assume three items, 1, 2, and 3, and two players, player A and player B. Player A has values of 10 for $\{1, 2\}$ and 100 for $\{2, 3\}$. Player B has values of 3 for $\{2\}$ and 49 for $\{3\}$.

In the initial round, the price of each item is 0. Player A declares $\{1, 2, 3\} = \{1, 2\} \cup \{2, 3\}$, and player B declares $\{2, 3\} = \{2\} \cup \{3\}$. The temporal price p_t is 0. Here, for item 1, only player A declared $\{1, 2\}$. This is the case of 3-(b). Thus, the baseline price for item 1 is decided as $p_t = 0$. Player A gets an option to buy item 1 at the price of 0. Then, the provisional price p_t continues to be increased ϵ.

When $p_t = 3$, player A declares $\{1, 2, 3\} = \{1, 2\} \cup \{2, 3\}$. Player B declares $\{3\}$. Since $p_t = 3$, the temporal price of bundle $\{1, 2\}$ is 6 since the size of $\{1, 2\}$ is 2. Also, the prices of $\{2, 3\}$, $\{3\}$, and $\{2\}$ are 6, 3, and 3, respectively. Player B does not declare $\{2\}$, since her utility on $\{2\}$ is $0 (= 3 - 3)$.

Here, for item 2, only player A declared $\{1,2\}$ or $\{2,3\}$. This is the case of 3-(b). Thus, the baseline price for item 2 is set as $p_t = 3$. Player A gets an option to buy item 2 at the price of 3. Then, the provisional price p_t continues to be increased ϵ. This continues until $p_t = 49$ in this case. We summarize this process in Table 1.

Table 1: Example

p_t	A's bid	B's bid	A's opt.	B's opt.
0	$\{1,2,3\}$	$\{2,3\}$	item 1	
3	$\{1,2,3\}$	$\{3\}$	item 2	
5	$\{2,3\}$	$\{3\}$		
49	$\{2,3\}$	$\emptyset$	item 3	

The auctioneer announces the baseline price for each item and the bidders who have options to purchase items. Here, player A has options to buy all items. This is step 4 in the protocol. Player A calculates the prices and the utilities for bundles. The baseline prices of options for items 1, 2, and 3 are 0, 3, and 49, respectively. According to definition 7, the prices of $\{1,2\}$ and $\{2,3\}$ are 6 and 98, respectively. Player A's utilities for $\{1,2\}$ and $\{2,3\}$ are $4(=10-6)$ and $2(=100-98)$. Thus, the auction is completed. Player A purchases $\{1,2\}$ at price 6.

Protocol

The open Ascending-price Combinatorial Auction based on AM-MB Protocol is designed as follows:

1. The auctioneer announces the current unit price, p_t (the initial unit price is 0). The temporal price $p_t(B)$ of bundle B is calculated based on $w_B \times p_t$.
2. Each bidder i is asked to declare a set of items B_i, where he might be interested in buying item $l \in B_i$ at the current price p_t. When bidder i uses a straightforward bidding strategy, i includes item l in B_i if for some $B \ni l$, where B is minimal and $v(B, \theta_i) > p_t(B)$.
3. (a) When $\forall i \neq j, B_i \cap B_j = \emptyset$, if the baseline price for item l has not been determined yet, we set the baseline price $p_{base}(l)$ for l to the current unit price p_t. Player i who declares $B_i \ni l$ gets an option to buy item l. Go to 4.
   (b) Otherwise, if there exists exactly one bidder i, such that $l \in B_i$ and the baseline price for item l has not been determined yet, we set the baseline price $p_{base}(l)$ for l to the current unit price p_t. Player i who declares $B_i \ni i$ gets an option to buy item l. Then, increasing ϵ on the unit price, go to 1.
4. The auctioneer announces the baseline price for each item and the bidders who have options to purchase items.
5. Each bidder declares the bundle that maximizes utility, giving him or her a positive utility, and this includes an item that the bidder has an option to buy. The price of a bundle is calculated based on the definition 7. Here, bidders who declare a bundle that is not $\emptyset$ are prohibited from purchasing nothing. This rule avoids stay-high bidding (discussed in "Avoiding Stay-high Bidding" in the next section).
6. Each bidder is allocated the bundle declared by him/her.

We call the above steps 1 to 3 a "round."

Features of the Protocol

Convergence on Baseline Price

Theorem 4 *In the proposed open ascending-price combinatorial auction we proposed , if ϵ is sufficiently small, and each bidder declares the baseline price $p_{base}(l)$ for each item converges straightforwardly into the baseline price $p_{base}(l)$ defined in definition 6.*

Proof: For any item l, we assume $p_t = p_{base}(l)$ holds in the k-th round (since ϵ is small enough, we can select such a k). We show that when $k' < k$, the auction does not finish. Furthermore, in the k-th round, at most one bidder declares that the bundle that includes l has a positive utility.

We derive a contradiction by assuming that at $k' < k$, (at most) one bidder declaring the bundle that includes l remains. In this round, $p_t < p_{base}(l)$ holds. Thus, in terms of $p_{base}(l)$, for two bidders j^* and $j^{2nd} = \arg\max_{j \neq j^*, B \ni l} v(j, B)/w_B$, where B is a minimal bundle, and for bundle B^* and bundle B^{2nd}, $v(j^*, B^*) > w_{B^*} p_t$ and $v(j^{2nd}, B^{2nd}) > w_{B^{2nd}} p_t$ hold.

At least two bidders, j^* and j^{2nd}, have positive utilities of bundle B^* and bundle B^{2nd}. Bundle B^* and bundle B^{2nd} include l. This contradicts the assumption.

When $k' = k$ holds, $p_t = p_{base}(l)$ holds. According to definition 6, it is obvious that at most one bidder declares that the bundle including l has a positive utility. □

Straightforward bidding is an ex-post Nash equilibrium

Here, we prove that straightforward bidding, where bidders bid truthfully in each iteration, constitutes an ex-post Nash equilibrium. Namely, if all bidders except bidder i bid truthfully, then buyer i has no incentive to follow any strategy other than the truth bidding strategy.

Assumption 1 *(Revealed preference activity rule) The revealed preference activity rule interprets bids in each round as placing constraints on a buyer's valuation, given a model of straightforward bidding, and ensures that bids in each round are consistent with some valuation function. Buyers must conform to the activity rule to remain active in the auction.*

Assumption 2 *(Proxy agents) Proxy agents follow straightforward bidding strategies on the basis of partial value information. Proxy agents ensure that bids in each round are consistent with some value function. Proxy agents must maintain consistency so that bidders cannot change their mind about their partial valuation if such a change would also change an earlier decision made by the proxy agent.*

Theorem 5 *If bidders obey the revealed preference activity rule in assumption 1 or employ proxy agents in assumption2, in the proposed open ascending auction, straightforward bidding is an ex-post Nash equilibrium.*

Proof: Fix the strategies of other bidders to straightforward bidding. The valuations are v_{-i} for all bidders except i. By bidding truthfully, buyer i receives payoff $\pi^{AM-MB}(v_i, v_{-i})$ from the equivalence between AM-MB payments and the open ascending auction payment. Consider some non-truthful strategy from buyer i that is consistent with $\hat{v}_i$ via the revealed preference activity rule or proxy agents. The open ascending auction will implement the outcome of the AM-MB auction for valuations $(\hat{v}_i, v_{-i})$. However, buyer i will always prefer the AM-MB outcome for valuations $(v_i, \hat{v}_{-i})$ because the AM-MB protocol (an instance of the PORF protocol) is strategyproof. Thus, in the proposed open ascending auction, straightforward bidding is an ex-post Nash equilibrium. □

Avoiding Stay-High Bidding

In step 5 of the protocol, bidders who declare a non-empty bundle are prohibited from purchasing nothing. Note that even if bidder i has an option to buy item l, bidder i can choose not to buy item l. Therefore, bidder i can choose not to buy anything even if i has one or more options. However, if bidder i declares $B_i \neq \emptyset$ at step 3 (a), then bidder i needs to buy $B \subseteq B_i$, where $B \neq \emptyset$.

Without this rule, as long as other bidders' strategies remain the same, the stay-high bidding strategy gives the same utility as the straightforward/sincere bidding strategy. With this rule, this is no longer true and using the stay-high bidding strategy becomes a risky proposition.

Low Communication Cost

One advantage of the AM-MB-based open ascending-price combinatorial auction is its low communication cost. In each round of the protocol, only a single price is announced to bidders, and each bidder declares only a single set of bundles. Bidders do not need to declare 2^m evaluation values in general combinatorial auctions, e.g., VCG.

Social Surplus and Revenue

In this section, we compare the AM-MB and M-MB protocols in terms of generated social surplus and revenue. We assume that each bidder has exaxtly one minimal bundle. This means that each bidder is single-minded. We say bidder i is single-minded if i requires only one bundle. Therefore, we determine the size of the minimal bundle by using an exponential distribution $d_e(k) = Ce^{-pk}$(Fujishima, Leyton-Brown, & Shoham 1999). By using this distribution, many small minimal bundles are created. The probability that a size k bundle is created is e^p times larger than that of a size $k+1$ bundle. The items included in the minimal bundle are randomly selected, and the evaluation value for the minimal bundle is randomly selected from $[0, k]$. This setting reflects the situation in which bidders tend to hope to purchase small bundles. For example, on the Internet, many unspecified people can attend an auction, and they tend to submit bids for different small bundles.

The experimental environment is written in JDK1.4.2 on MacOS 10.3 and a PowerPC G4/dual machine. To compute VCG payments, we employ several pruning methods, e.g., BINs and dominant sets pruning, with the CASS method(Fujishima, Leyton-Brown, & Shoham 1999).

Figure 1 and Figure 2 show experimental results. The number of items are 7. There are 2^7 possible bundles. We created 1,000 different problems and show the averages of the social surplus and seller's revenue by varying the number of players. Both the proposed open ascending auction and AM-MB protocols can achieve equivalent social surplus and revenue.

In Figure 1, the vertical axis shows the ratio of social surplus compared with the optimal social surplus that can be calculated by VCG. "M-MB SS" and "AM-MB SS" mean the M-MB protocol's social surplus and the AM-MB protocol's social surplus, respectively. The horizontal axis shows the number of players. AM-MB can attain 85 to 95 % of the efficient social surplus. Furthermore, AM-MB's social surplus is almost always larger than M-MB's social surplus. In particular, when the number of players is 20 to 40, AM-MB can work better than M-MB. This is because M-MB tends to overestimate the prices of bundles compared with AM-MB. AM-MB carefully calculates the bundle prices as the bundle size times the average evaluation value for each good in the bundle and then chooses the maximum price. M-MB simply uses the bundles' valuation values as the bundle prices and chooses the maximum price. Here, the social surplus decreases after 40 players, since we assume single-minded players. As the number of players increases, so too does the possibility of adequate allocation. Namely, while AM-MB can succeed in making adequate allocations in the early stage, in terms of the number of players, VCG and M-MB can succeed after 40 players.

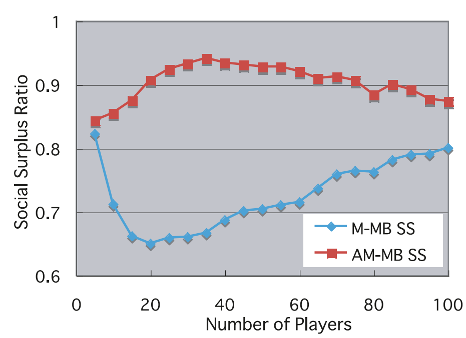

Figure 1: Experimental Results: Social Surplus

In Figure 2, the vertical axis shows the ratio of revenue compared with the revenue calculated by VCG. "M-MB RE" and "AM-MB RE" mean the M-MB protocol's revenue and the AM-MB protocol's revenue, respectively. The horizontal axis shows the number of players. As with the case of social surplus, when the number of players is 20 to 40, AM-MB can work better than M-MB. This is because the probability of a successful allocation increases with the number of players. Furthermore, both AM-MB and M-MB can gain more revenue than VCG. This is because both AM-MB and M-MB overestimate the prices compared with VCG.

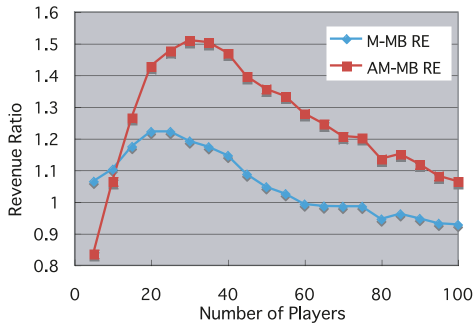

Figure 2: Experimental Results: Revenue

AM-MB's revenue is almost always larger than M-MB's revenue. At the maximum, AM-MB's revenue is 1.25 times as large as M-MB's revenue. Namely, AM-MB increases the social surplus and revenue over those values of M-MB when players are single-minded and tend to prefer small bundles.

Improving Social Surplus

For the case of two bidders A and B with bundles as $\{1, 2\}$ and $\{2, 3\}$ and both the valuations being 10, the protocols proposed above does not allocation item 2. To increase the social surplus, we can slightly modify the protocol so that both bidders can obtain the option. A similar modification can be applied to the AM-MB protocol.

In the improved AM-MB protocol, bundle B that maximizes player i's utility (that includes 0) is allocated to player i. In the improved open ascending protocol, we can improve the following three points: (1) Each player declares his demand which is an union set of bundles that give non-negative utility to him. (2) Suppose two or more bidders declare demands for one item in round r. If the demand decreases to 0 in the next round $r + 1$, the improved protocol gives options to the bidders who declared demands in the round r. (3) Each bidder selects a bundle that maximizes his utility (that includes 0). Overlaps among bidders can be happened when their utilities are 0. In this case, the improved protocol randomly select one bidder to assign a bundle.

Related Work

Our AM-MB-based open ascending-price combinatorial auction provides low computational cost without optimization. Moreover, straightforward bidding is an ex-post Nash equilibrium. No existing protocol offers both of these characteristics. The following works are open ascending-price combinatorial auctions that have other objectives: iBundle (Parkes & Ungar 2000) was efficient for straightforward bidder strategies, but only those for which straightforward bidding was not in equilibrium. iBundle Extend & Adjust (iBEA)(Parkes & Ungar 2002) extended iBundle to implement the outcome of VCG. iBEA needs to solve winner determination problems in each round. Another paper (Iwasaki, Yokoo, & Terada 2005) proposed an open ascending-price multi-unit auction called AOP that is robust against false-name bids. In Ausubel's ascending-price proxy auction, when bidders employ sincere bidding proxies, the results converge into a core (Ausubel & Milgrom 2002).

Conclusions

In this paper, we proposed the AM-MB protocol and extended it to an open ascending combinatorial auction. The AM-MB protocol is strategyproof while being computationally inexpensive because of its greedy allocation method. The experimental results show that the AM-MB protocol can obtain higher social surplus and revenue than the M-MB protocol. As future work, we intend to investigate how effectively the AM-MB protocol can be applied to situations in which the expertise of bidders in assessing their valuations for goods is asymmetric.

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